PROPOSITIONAL LOGIC: SEMANTICS

3

# Do Conditionals Have Truth-Conditions?

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## 1. INTRODUCTION

In the first part of this paper (Sects. 2 and 4) I rule out the possibility of truth-conditions for the indicative conditional 'If A, B' which are a truth-function of A and B. In the second part (Sect. 6) I rule out the possibility that such a conditional has truth-conditions which are *not* a truth-function of A and B; I rule out accounts which appeal, for example, to a stronger-than-truth-functional 'connection' between antecedent and consequent, which may or may not be framed in terms of a relation between possible worlds, in stating what has to be the case for 'If A, B' to be true. I conclude, therefore, that the mistake philosophers have made, in trying to understand the conditional, is to assume that its function is to make a statement about how the world is (or how other possible worlds are related to it), true or false, as the case may be. Along the way (Sects. 3 and 5) I develop a positive account of what it is to believe, or to be more or less confident, that if A, B, in terms of which an adequate logic of conditionals can be developed. The argument against truth-conditions is independent of this positive account of the conditional, as I show that any truth-conditional account has counterintuitive consequences, as well as clashing with my positive thesis. But the positive account prevents the essay from merely having created a paradox, or a vacuum.

The essay is inspired by Ernest Adams's book, *The Logic of Conditionals*.[1] My positive thesis is a less technical variant of his. He proves

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1. Ernest Adams (1975). Some historical background: Robert Stalnaker (1970) was, I believe, the first to suggest that insight into the semantics of conditionals might be gained from the probability theorist's notion of a conditional probability, P(B/A) (the probability of B given A). Judgements about how probable it is that if A, B, seem to coincide with judgements about the





the negative result too, but hardly perspicuously. My aim, in trying to extract an intuitively compelling argument from a somewhat baffling piece of algebra, is not only to make this way of thinking about conditionals more widely, and more deeply, appreciated. It is also, by weakening the assumptions, to provide a stronger proof of the negative result. I hope to render the positive thesis more plausible, too, by presenting it less technically.

It should not need emphasis that in the conditional we have an indispensable form of thought, which plays a large part in both theoretical reasoning about what is the case and practical reasoning about what to do. Its basic role may be described thus: we are not omniscient; we do not know as much as it would be useful for us to know. We are constantly faced with a range of epistemic possibilities—things that, as far as we know, may be true, when the question whether they are true is relevant to our concerns. As part of such practical or theoretical reasoning, it is often necessary to *suppose* (or assume) that some epistemic possibility is true, and to consider what else would be the case, or would be likely to be the case, given this supposition. The conditional expresses the outcome of such thought processes. It is worth remembering that any type of speech act can be performed within the scope of a supposition. There are conditional questions, commands, etc., as well as conditional assertions.

> If he phones, what shall I say?
> If I'm late, don't stay up.
> If you're determined to do it, you ought to do it today.

To assert or believe that if A, B is to assert (believe) B within the scope of the supposition, or assumption, that A.[2] This is bland

probability of B given A. Stalnaker suggested that we should define the conditional as that proposition whose probability is so measured. David Lewis (1976) was the first to prove that there is no such proposition. As a result, Stalnaker and Lewis rejected the equation of the probability of a conditional with a conditional probability, the former defending a non-truth-functional account, the latter the truth-functional account of indicative conditional propositions. Adams, instead, retains the equation, and denies that the conditional is, strictly speaking, a proposition. In this essay I support Adams. I am also indebted, in the proof of sect. 6, to I. F. Carlstrom and C. Hill's review of Adams (1975) in *Philosophy of Science* (1978).

2. I take this formulation from Mackie (1973), ch. 4. Mackie had the right idea, but did not have adequate arguments for his rejection of truth-conditions.

enough, it would seem, to be not worth denying. Now, from a truth-conditional perspective, this double illocutionary force—an assumption, and an assertion within its scope—is eliminable—is reducible to, or equivalent to, a plain assertion. If conditionals have truth-conditions, to assert 'If A, B' is to assert that its truth-conditions obtain. One way of presenting the conclusion of this essay, then, is that the double illocutionary force is *in*eliminable; there is no proposition such that asserting *it* to be the case is equivalent to asserting that B is the case given the *supposition* that A is the case. For any proposed truth-condition, I shall show that there are epistemic situations in which there is a divergence between assent to the proposition with that truth-condition and assent to the conditional.

The main argument of the essay concerns indicative conditionals. The thesis extends to subjunctive or counterfactual conditionals, but I shall not have space to argue that here.[3] The distinction, from the present perspective, is not between two types of conditional connection, but between two types of supposition, or better, two kinds of context in which a supposition is made. One can suppose that A, taking oneself to know that not-A; and one can suppose that A, not taking oneself to know that not-A. Typically, the subjunctive or counterfactual conditional is the result of the first kind of supposition, the open or indicative conditional the result of the second kind. An apparent difficulty which actually clarifies the point: I take myself to know that the carpet I am now looking at is not red. I may say 'If it had been red, it would have matched the curtains.' But I may also say 'If it *is* red—well, I have gone colour-blind or am suffering some sort of delusion'. In the subjunctive, I am taking it for granted that I am right in thinking it is not red. In the indicative, I am supposing that I am wrong. I am considering it to be an epistemic possibility that it is red, despite appearances. The importance of this for present purposes is that the positive account of indicative conditionals to follow assumes that the antecedent is always treated as epistemically possible by the speaker. When that condition is not satisfied, the conditional will be treated as a subjunctive, in the extension of the thesis. It will not matter if this distinction between two kinds of supposing does not match perfectly the grammatical distinction. It is enough if any conditional thought can be explained in one of the two envisaged ways.

One further remark about the methodology of this essay: while it is no part of my purpose to deny that some conditionals are certain, on a priori or other grounds, the argument hinges upon the undeniable

3. See Adams (1975), ch. 4. More support for a unified theory of indicative and counterfactual conditionals is found in Ellis (1978 and 1984).



fact that many conditionals, like other propositions, are assented to or dissented from with a degree of confidence less than certainty. We are frequently uncertain whether if A, B, and our efforts to reduce our uncertainty often terminate, at best, in the judgement that it is probable (or improbable) that if A, B. Of course, the truth-conditions theorist does not have to deny these undeniable facts. For him, to judge it more or less probable that if A, B is to judge it more or less probable that its truth-conditions obtain. But this pinpoints his mistake. I show that uncertainty about a conditional is not uncertainty about the obtaining of any truth-conditions. If a conditional had truth-conditions, it would be. Therefore, a conditional does not have truth-conditions. That is the structure of the argument to follow.

## 2. THE TRUTH FUNCTIONAL ACCOUNT

There are sixteen possible truth-functions of A and B. Only one is a candidate for giving the truth-conditions of 'If A, B'. Indeed, the following two assumptions are sufficient to prove that *if* 'If A, B' is truth-functional, it has the standard truth-function (that is, it is equivalent to '~(A & ~B)' and to '~A ∨ B'). (1) 'If P & Q then P' is true, whatever the truth-values of P and of Q; (2) Sentences of the form 'If A, B' are sometimes false, i.e. are not all tautologies. So we may safely speak of *the* truth-functional account.

It is important to recognize that there are powerful arguments in favour of the truth-functional account. Here are two. First, take any two propositions, B and C. Information that at least one of them is true seems sufficient for the conclusion that if C is not true, B is true. The converse inference is uncontroversial.[4] Let C be ~A, and we appear to have vindicated the equivalence between '~A ∨ B' and 'If A, B'. Second, information that A and C are not both true seems to license the inference that if A is true, C is not. Again, the converse implication is uncontroversial. Let C be ~B, and we appear to have vindicated the equivalence between '~(A & ~B)' and 'If A, B'. (I shall show later that my positive account will preserve the force of these arguments, while no account in terms of non-truth-functional truth-conditions can.)

But alas, there are well known difficulties for the truth-functional account: ~A entails ~(A & ~B), for any B. B entails ~(A & ~B), for any A. So, according to this account,

The Labour Party will not win the next election

4. Suppose that if C is not true, B is true. Then, either C is true or (it isn't, in which case) B is true.

entails

> If the Labour Party wins the next election, the National Health Service will be dismantled by the next government.

Anyone who accepts the former and rejects the latter is (on this account) inconsistent.

Similarly,

> The Conservative Party will win the next election

entails

> If a horrendous scandal emerges during the campaign involving the Prime Minister and most of the Cabinet, the Conservative Party will win the next election.

Again, anyone who accepts the former and rejects the latter has, on this account, inconsistent beliefs.

H. P. Grice (1975) argued that the truth-functional account can withstand these objections, provided that we are careful to distinguish the false from the misleading but true. There are many ways in which one can speak the truth yet mislead. One way is to say something weaker than some other relevant thing one is in a position to say. Consider disjunctions. I am asked where John is. I firmly believe he is in the bar, and I know that he never goes near libraries. Inclined to be unhelpful but not wishing to lie, I say

> He is either in the bar or in the library.

I could go on: or at the opera or at the church or . . .)

My hearer naturally concludes that this is the most precise information I am in a position to give, and also concludes from the truth (let us assume) that I told him

> If he's not in the bar he is in the library.

The conditional, like the disjunction, according to Grice, is true provided that he's in the bar, but misleadingly asserted on these grounds.

I shall now show that this defence of the truth-functional account fails. Grice drew our attention to the existence of propositions which



a person *has grounds to believe true* but which it would be unreasonable, in normal contexts, to assert. A contrast is invoked between what one may reasonably *believe* and what one may reasonably *say*, given one's grounds. I do not dispute that it is important to recognize this phenomenon. It does, I think, correctly explain the behaviour of disjunctions. Being sure that John is in the bar, I cannot consistently *disbelieve* the proposition 'He is either in the bar or in the library'; indeed, if I have any epistemic attitude to that proposition, it should be one of belief, however inappropriate it is for me to assert it.

A good enough test of whether the Gricean story fits the facts about disjunctions is this: I am asked to respond, 'Yes', 'No', or 'No opinion', to the disjunction. Being sure of one disjunct, I should surely answer 'Yes'.

Here there is a striking contrast between disjunctions and conditionals. Imagine an opinion poll shortly before an election. Again, the subject is asked to respond 'Yes' if he thinks a proposition true, 'No' if he thinks it false, 'No opinion' otherwise. The subject is honest and prides himself on his consistency. Here are some of his responses:

1. The Labour Party will win (L) . . . . . . . . . . . . . . . . No
2. The Labour Party won't win (~L) . . . . . . . . . . . . . . . . Yes
3. Either the Labour Party won't win or ____ (~L ∨ X) (Fill in the blank as you will: If he accepts that (2) is true, he must, if rational, accept that at least one of two propositions, of which (2) is one, is true.) Yes
4. If the Labour Party wins, the National Health Service will be dismantled by the next government (If L, N) . . . . . . . . . . . . No

Now, on the truth-functional account, this person has blatantly inconsistent beliefs. His saying 'Yes' to (2) and 'No' to (4) is on a par with someone's saying 'Yes' to 'It's red and square' and 'No' to 'It's red'. The parallel is exact, for, on the truth-functional account, to deny (4) is equivalent to accepting L & ~N; he cannot consistently accept this yet deny L. But it is surely quite clear that our subject, in accepting (2) and rejecting (4), is not contradicting himself.

In the case of disjunctions, the predicted Gricean contrast between what it is reasonable to believe and what it is reasonable to say, given one's grounds, is discernible. In the case of conditionals, it is not. (I do not mean that the distinction does not apply to conditionals, but that it fails as a defence of the truth-functional account.) The purpose of the opinion poll is simply to elicit someone's opinions, irrespective of whether they would constitute appropriate remarks in an ordinary conversational interchange. We can stipulate that the subject is honest and serious. We must either accuse him of gross inconsistency, or accept that the conditional is not truth-functional.

This case against the truth-functional account cannot be made in terms of beliefs of which one is *certain.* Someone who is 100 per cent certain that the Labour Party won't win has (on my account of the matter) no obvious use for an *indicative* conditional beginning 'If they win'. But someone who is, say, 90 per cent certain that they won't win can have beliefs about what will be the case if they do. The truth-functional account has the immensely implausible consequence that such a person, if rational, is at least 90 per cent certain of any conditional with that antecedent.

The principle I am appealing to is this:

> If A entails B, it is irrational to be more confident of A than of B.

For instance, it is irrational to be more confident that a thing is red than that it is coloured. If the entailment is one-way, any way of rendering A true renders B true, but not conversely. B may be true when A is not. B has more chance of being true than A.[5]

Given that some entailments are exceedingly complex, the principle, in its full generality, no doubt has the consequence that no one is fully rational. But here we are dealing with a simple, decidable, truth-functional entailment of the most basic kind. If the truth-functional account were correct, it would be a straightforward matter to get the subject to recognize that he has inconsistent beliefs.

## 3. WHAT IT IS TO JUDGE THAT IF A, B

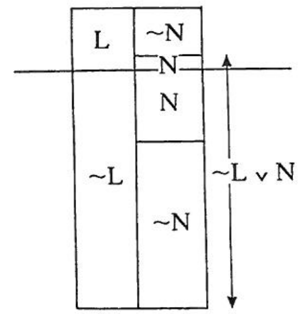
The critique of the truth-functional account has yet to be completed, but it is useful here to introduce, by way of contrast, my positive account of the consistent judgements our subject *is* making when he accepts (2) and rejects (4). Figure 1 is a diagrammatic representation of how likely he considers the various possibilities, L, ~L, N, ~N, L & N, L & ~N, etc., to be, vertical height representing probability. In considering whether if L, N, the subject assumes L; that is, he ignores the ~L-possibilities, the lower part of the diagram. Considering just those possibilities above the wide line, he asks how likely it is that N. Answer: very unlikely. On the other hand, he is committed to

5. The principle is provable in probability theory: writing '$\leftrightarrow$' for logical equivalence, $B \leftrightarrow (A \,\&\, B) \vee (\sim A \,\&\, B)$. So $P(B) = P(A \,\&\, B) + P(\sim A \,\&\, B)$. If A entails B, $A \leftrightarrow A \,\&\, B$. So $P(B) = P(A) + P(\sim A \,\&\, B) \geq P(A)$.



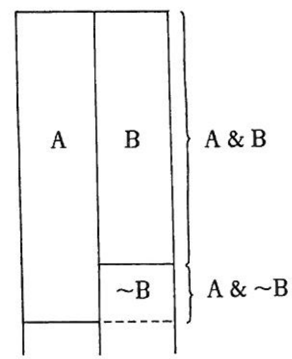
believing $L \supset N$, that is $\sim L \vee N$, to be slightly more probable than ~L, that is very likely.

FIG. 1




To judge it probable that $A \supset B$ is to judge it improbable that A & ~B. To judge it probable that if A, B is not only to judge it improbable that A & ~B, but to judge this to be less probable than A & B. 'Is B likely given A?' is the question 'Is A & B nearly as likely as A?' (see Figure 2).

FIG. 2




That A & ~B be small, which is necessary and sufficient for the conditional to be probable on the truth-functional account, is necessary but not sufficient on this account. If A & ~B is large, greater than ½, say, there isn't room for A & B to be larger still. However, A & ~B can be small and A & B smaller still, as in the original example. In such a case, the material implication is probable but the conditional is not.

A simple example of the contrast between the two accounts: How likely is it that if this (fair) die lands an even number, it will land six? On my approach, we assume that the die lands an even number; given that assumption, there are three equal possibilities, one of which is six. So the answer is ⅓. On the truth-functional approach, the answer is

⅔: if the die lands not-even or six, that is, if it lands, 1, 3, 5, or 6, the conditional is true. So the conditional has four chances out of six of being true.

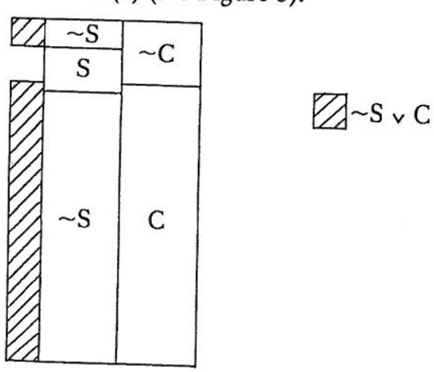
## 4. THE CASE AGAINST TRUTH FUNCTIONALITY CONTINUED

Let us continue our questionnaire to consider the second paradox of material implication:

5. The Conservative Party will win (C) ........................ Yes
6. Either ____ or the Conservative Party will win (__ ∨ C) (Fill in the blank as you like.) ................................ Yes
7. If a horrendous scandal emerges involving the Prime Minister and most of the Cabinet, the Conservative Party will win (If S, C) .... No

Such answers are not inconsistent. I grant that someone who is 100 per cent certain that the Conservatives will win will accept any conditional with an antecedent which he takes as an epistemic possibility and C as consequent. But that is not enough to prove the validity of the inference from C to 'If S, C'. Suppose our subject is 90 per cent certain that the Conservatives will win. He allows that they may not win, and that if certain, in his view unlikely, things happen, they will not win. So it is consistent to have a high degree of confidence that C and a low degree of confidence that if S, C. On the truth-functional account, this is, again, logically on a par with being very confident that it's red and square but very unconfident that it's square. On the other hand, his high degree of confidence in (5) does constrain him to at least that degree of confidence in (6) (see Figure 3).

FIG. 3




I said that the Gricean defence depends on a contrast between when a conditional is fit to be believed and when it is fit to be



asserted. I have shown that the conditions under which a conditional is believed do not fit the truth-functional account. So this defence fails. Frank Jackson (1979; 1980–1) defends the truth-functional account differently. His thesis is that for a conditional to be assertable, it must not only be believed that its truth-conditions are satisfied, but the belief must be *robust* or *resilient* with respect to the antecedent. This means that one would not abandon belief in the conditional if one were to discover the antecedent to be true. This ensures that an assertable conditional is fit for *modus ponens.* This condition is not satisfied if one believes A ⊃ B solely on the grounds that ~A. If one discovered that A, one would abandon one's belief that A ⊃ B, rather than conclude that B. I think this defence is open to the same objections as Grice's. There is simply no evidence that one *believes* a conditional whenever one believes the corresponding material implication, and then is prepared to *assert* it only if some further condition is satisfied.

I have been assuming that if a sentence is correctly assigned certain truth-conditions, a competent speaker believes that sentence if and only if he believes these conditions are fulfilled; and, provided that he is honest and has no wish to hide his opinion, will say so if asked 'Do you believe that A?' It may be objected that the distinction between its truth-conditions and other aspects of a sentence's use is more a theorist's, less a practitioner's distinction than I have allowed. If this is so, then we must ask, what theoretical purpose is served by the assignment of these truth-conditions? To explain the validity of inferences? But it does this very badly. I have shown this for the two simplest types of example, but these generate indefinitely many other counterintuitive 'valid' inferences. Here is a new 'proof' of the existence of God:[6] 'If God does not exist, then it is not the case that if I pray my prayers will be answered (by Him). I do not pray. (So it *is* the case that if I pray . . .) So God exists'. The extent to which the truth-functional account succeeds in capturing the validity of inferences is explained by the fact that the material implication is essentially weaker than the indicative conditional (see above) and so is the extent to which it fails.

Another suggestion is that the truth-functional account explains the behaviour of embedded conditions: it explains the contribution of the truth-conditions of 'If A, B' to those of '(If A, B) or (if C, D)', for example. But, unsurprisingly, the truth-functional account yields counterintuitive results for sentences containing conditionals as constituents. For example, it tells us that the following is a tautology:

6. I owe this example to W. D. Hart.

(If A, B) or (if not-A, B).

So anyone who rejects the first conditional must, on pain of contradiction, accept the second. So if I reject the conditional 'If the Conservatives lose, Thatcher will resign', I am committed to accepting 'If the Conservatives win, Thatcher will resign'![7]

We have not been able to find any theoretical purpose well served by these truth-conditions. There does not appear to be any indirect evidence in its favour to mitigate against the direct evidence against it—the fact that belief in a conditional and belief in a material implication do not coincide.

## 5. THE POSITIVE ACCOUNT CONTINUED

I outlined my positive account of belief in a conditional in Sect. 3. In considering how likely it is that if A, B, one assumes A, that is, ignores the possibility that ~A. Relative to that assumption, one considers how likely it is that B (see Figure 2). This yields the following criterion:

> X believes that (judges it likely that) if A, B, to the extent that he judges that A & B is nearly as likely as A
> or, roughly equivalently, to the extent that he judges A & B to be more likely than A & ~B.

If we were to make the idealizing assumption that a person's subjective probability judgements are precise enough to be assigned numbers between one and zero inclusive, we could be more precise and say that the measure of X's degree of confidence in the conditional 'If A, B' is the ratio

$$\frac{P_x(A \& B)}{P_x(A)}$$

This ratio is known in probability theory as *the conditional probability of B given A.* Our positive thesis could be stated, then

7. Lewis (1976) gives as his reason for rejecting the no-truth-conditions view that it cannot explain embedded conditionals. He goes on to defend the truth-functional account, attempting to explain away some of its paradoxical features. But he does not address the problem that the truth-functional account gives absurd results for embedded conditionals.



> A person's degree of confidence in a conditional, if A, B, is the conditional probability he assigns to B given A.

However, my argument does not depend upon the idealizing assumption of precise numerical values. Also, even if we grant numerical values, the ratio must not be taken as a reductive definition of the conditional probability, as though one first had to ascertain how probable it is that A and that A & B, and then divide the second by the first. Typically, one does not have to decide how likely it is that A in order to judge that B is likely given A. I may have given no thought to the matter of how likely it is that the Labour Party will win yet be confident that if they win public spending will increase; this latter confidence entails confidence that, however, likely it is that they win, it is nearly as likely that (they win and public spending increases). The non-reducibility is particularly obvious when, as part of some practical reasoning, one considers conditionals of the form 'If I do *x*, such-and-such will happen.' It would be absurd to hold that I have to know how likely it is that I will do *x* before I can assess such a conditional.

Let us consider some special cases. If I am certain of a conditional, for example that if he is a bachelor, he is unmarried, then, however likely it is that he is a bachelor, it is equally likely that he is a bachelor and unmarried. The ratio is 1. Given a conditional in which I have the lowest possible degree of confidence, for example, that if he's a bachelor, he's married, I assign probability 0 to the conjunction of antecedent and consequent, and hence to the ratio. If I think it is 50:50 that if you toss this coin, it will land heads, then, whatever the probability that you toss it, the probability that (you toss it and it lands heads) is half as much: the ratio is 1:2.

This measure has the advantage of allowing the probability of the conditional to be independent of the probability of the antecedent. On the truth-functional account, the probability that if you toss the coin it lands heads depends crucially on how probable it is that you toss it. Suppose it is much less likely now that you toss the coin than it was a minute ago. The probability of the material implication, which is equivalent to:

> Either you won't toss it, or (you will and it will land heads)

has greatly increased. But the probability of the consequent on the assumption that the antecedent is true has remained the same.

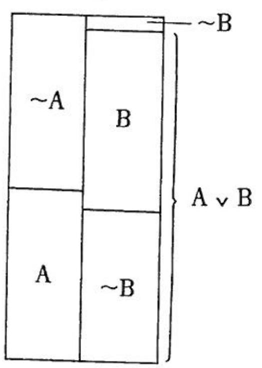
Non-truth-functional accounts of the truth-conditions of conditionals demand some sort of 'strong connection' between antecedent and consequent for the conditional to be true. Such a connection is clearly lacking in

If you toss this (fair) coin, it will land heads.

On such accounts, the conditional is then certainly false. It should have probability 0. But surely, if someone is told 'the probability is 0 that if you toss it it will land heads', he will think it is a double-tailed or otherwise peculiar coin. Keeping the structure but changing the content of the example—a dog either bites or cowers when strangers approach, apparently at random, and with about equal frequency of each. Could one in good faith tell a stranger that the probability is zero (i.e. it is certainly false) that if he approaches, the dog will bite?

I think I have said enough to render plausible the claim that the measure of acceptability of a conditional 'If A, B' is the conditional probability of B given A. Without idealizing, the basic thesis that to assess how probable it is that if A, B, one assumes A, and considers how probable it is that B, under that assumption; and that that thought process is equivalent to considering whether A & B is nearly as likely as A. More evidence for the thesis comes from considering which inference-patterns involving conditionals are valid. There is not space to present this evidence fully,[8] but I shall end this section by saying something about the inference from 'A ∨ B' to 'If not-A, B'. As I said at the beginning of Sect. 2, if this inference were valid, the truth-functional account would be correct. And the inference appears very plausible. We shall see how to explain these facts.

FIG. 4




If I am agnostic about A, and agnostic about B, but confident that A or B, I must believe that if not-A, B. (See Figure 4. If in almost all

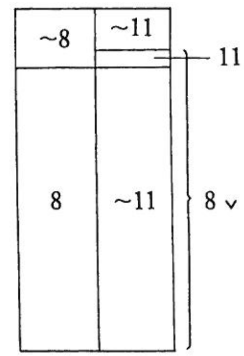
8. See Adams (1975), ch. 1.



possibilities, either A or B is true; and A and B are each true in approximately half the possibility-space; then in almost all not-A possibilities, B is true.) This is the normal situation in which a belief that A or B will play an active role in my mind, as a premiss or as anything else, for example, someone has told me that A or B, or I have eliminated all but these two possibilities.

On the other hand, if my belief that A or B derives solely from my belief that A, the inference is not justified. For example, I wake up and look at the clock. It says eight o'clock. It is fairly reliable but by no means infallible. I am 90 per cent confident that it is eight o'clock (within whatever degree of precision with which we make such statements). So, were I to consider the matter, I must be at least 90 per cent confident that it is either eight o'clock or eleven o'clock. But this gives me no grounds for confidence that if it is not eight, it is eleven (see Figure 5).

FIG. 5




As it is rare and rather pointless to consider disjunctions in circumstances such as these, it is not surprising that we mistake 'A or B; therefore, if not-A, B' for a valid argument.

## 6. THE CASE AGAINST NON-TRUTH-FUNCTIONAL TRUTH CONDITIONS

If a conditional has truth-conditions, the probability of a conditional is the probability that those conditions obtain. Suppose that a conditional has truth-conditions which are not a truth-function of its antecedent and consequent. This means that the number of logically possible combinations of truth-values of A, B, 'If A, B' is between five and eight. That is, at least one and at most all four possible combinations of truth-values for A and B split(s) into two possibilities: 'If A, B' true; 'If A, B' false. At most three of the following eight combinations of truth-value can be ruled out a priori:

| | A | B | If A, B |
|---|---|---|---|
| 1a | T | T | T |
| 1b | T | T | F |
| 2a | T | F | T |
| 2b | T | F | F |
| 3a | F | T | T |
| 3b | F | T | F |
| 4a | F | F | T |
| 4b | F | F | F |

What follows is a 'tetralemma'. I shall now show that wherever truth-functionality is assumed to fail, there are consequences incompatible with the positive thesis about the acceptance of a conditional; and that where there is a clash, intuition continues to favour the positive thesis rather than the non-truth-functional truth-conditions thesis.

First, suppose

> Assumption 1: A conditional has truth-conditions which are not truth functional when A and B are both true.

Thus 1a and 1b are two distinct possibilities. On this assumption, 'If A, B' would be like 'A before B' and 'A because B'. For example, the truth of 'John went to Paris' and of 'Mary went to Paris' leaves open the question whether 'John went to Paris *before* Mary went to Paris' is true; its truth depends on more than the truth-values of its constituents.

Consequence of Assumption 1:

> $C_1$: Someone may be sure that A is true and sure that B is true, yet not have enough information to decide whether 'If A, B' is true; one may consistently be agnostic about the conditional while being sure that its components are true (as for 'A before B').

This consequence is central to my argument. I pause to clarify and defend it. It does not *quite* follow *merely* from the assumption of non-truth-functionality. There are exceptions to claims of the same form. But the exceptions are special cases, which do not cast doubt on the case of conditionals.



First exception: take the operator 'It is self-evident that . . .'. 'It is self-evident that A' is not a truth-function of A when A is true. But it does not follow that one may be sure that A yet agnostic about whether it is self-evident that A. For there is no room for uncertainty about propositions of this last form. However, such an operator clearly contrasts with the operators, 'if', 'before', 'because', which, in general, make contingent a posteriori claims, about which there is plenty of room for uncertainty. Of course there are self-evident conditionals, such as 'If he's a bachelor, he's unmarried'; but they owe their self-evidence to the particular contents of the constituent propositions. They are not self-evident just because of the meaning of 'if'.

It could be objected that my argument, resting on $C_1$, will not have shown that those conditionals which *are* self-evident don't have truth-conditions. But this would be to claim that 'if' is ambiguous: that it has a different meaning in 'If he's a bachelor he's unmarried' and 'If John is in Paris, so is Mary.' I see no grounds for an ambiguity. My positive thesis has the consequence that self-evident conditionals are certain—the consequent is certain on the supposition that the antecedent is true; and that conditionals about which one may be uncertain cannot be understood in terms of truth-conditions. It offers a unified account of indicative conditionals which is incompatible with a unified account in terms of truth conditions. Unified accounts are prima facie preferable to accounts which postulate ambiguities. In the absence of a strong case for ambiguity, then, my argument still applies to all conditionals.

A second counter-example to the general claim about non-truth-functionality I owe to Raúl Orayen: Interpret 'A*B' as 'I am sure that A and sure that B'. This is not a truth-function of A and B when A and B are both true. But it does not follow that I can be sure that A and sure that B yet agnostic about A*B. It could be replied that, as we do not have incorrigible access to our own beliefs, it *is* possible to be sure that A, sure that B, yet unsure about whether one is sure, i.e. unsure about A*B.[9] But in any case, any putative truth conditions of 'If A, B' will surely be unlike those of 'A*B' in being independent of the state of mind of any one individual. The hypothesis under consideration, Assumption 1, is that the truth of A and of B is insufficient to determine the truth of 'If A, B'. One doesn't have to be an extreme realist about truth to insist that whatever else is necessary is *in general* nothing to do with one individual's epistemic state. I say 'in general' because, as before, there will be special cases—conditionals which are

9. I owe this point to Raymundo Morado.

*about* the state of mind of some one individual; and *perhaps* to some of these, the individual concerned has incorrigible access. But, to repeat, we are in the business of interpreting 'If' for all conditionals. The contribution it makes to the (alleged) truth conditions of sentences in which it occurs makes no reference to my state of mind—though in special cases, the A or the B in 'If A, B' may do so.

$C_1$ still stands, then. Now $C_1$ is incompatible with our positive account. Being certain that A and that B, a person must think A & B is just as likely as A. He is certain that B on the assumption that A is true.

$C_1$ also conflicts with common sense. Admittedly, the conditional 'If A, B' is not of much interest to someone who is sure that both A and B are true. But he can hardly doubt or deny that if A, B, in this epistemic state. Establishing that the antecedent and consequent are true is surely one incontrovertible way of verifying a conditional. If you deny that if A, B, and I know that A and B are both true, I am surely in a position to correct you.

Assumption 1 must, then, be rejected. Truth-functionality cannot fail when A and B are both true. 'A & B' is sufficient for 'If, A, B'. Putative possibility 1b does not exist. We proceed to the second stage of the argument.

> Assumption 2: A conditional has truth-conditions which are not truth-functional when A is true and B is false.

Consequence of Assumption 2:

$C_2$: Someone may be sure that A is true and sure that B is false yet not have enough information to settle whether 'If A, B' is true, and hence be agnostic about the latter.

As with $C_1$, this is incompatible with our positive account, and also with common sense. Such a person knows enough to reject the claim that B is true on the assumption that A. 'A & ~B' is sufficient to refute 'If A, B'. Assumption 2 is false. Putative possibility 2a does not exist.

We have shown, then, that if a conditional has truth conditions, they are truth-functional for the two cases in which A is true. We shall now consider the cases in which A is false.

> Assumption 3: A conditional has truth-conditions which are not truth-functional when A is false and B is true.



Now suppose someone is sure that B but is uncertain whether A. On our positive account, he knows enough to be sure that if A, B: If B is certain, A & B is just as probable as A. This also accords with common sense. But according to Assumption 3, there are three possibilities—three ways the world may be—compatible with his knowledge:

| A | B | If A, B |
|---|---|---|
| T | T | T |
| F | T | T |
| F | T | F |

(I rely on the fact that we have established truth-functionality for the top line.)

A may be false, and if it is, some further condition has to be satisfied for 'If A, B' to be true, and he may not know whether it is satisfied. According to Stalnaker (1968), for instance, the further condition is that B be true in the closest possible world to the actual world in which A is true. And he might not know enough about the actual world to know whether this is so.

An example might help. I complain to John that he has not replied to my letter. He says he did—he posted the reply some weeks ago. I am not sure whether to believe him. Let A be 'He posted the reply' and B be 'I didn't receive it.' Our positive account has it that B is certain on the assumption that A, and so does common sense. But by Assumption 3, I should reason like this: 'I didn't receive the letter. Suppose he posted it: then the conditional is true. But suppose he didn't post it: this, together with the fact that I didn't receive it, is not sufficient for the conditional. It depends (say) on whether in the closest possible words in which he *did* post it, I still didn't receive it. And I can't be sure of that.'

Assumption 3, then, is incompatible with our positive account, and once more, intuition vindicates our account. Assumption 3 must be rejected. Putative possibility 3b does not exist.

> Finally, Assumption 4: Truth-functionality fails when 'A' and 'B' are both false.

Now consider someone who is sure that A and B have the same truth-value, but is uncertain which. For example he knows that John and Mary spent yesterday evening together, but doesn't know whether they went to the party. According to our positive account and according to common sense, he knows enough to be sure that if John went to the party (J), Mary did (M). (J & M is as likely as J; M is certain on the assumption that J.) But according to Assumption 4, he has to consider three possibilities compatible with his knowledge:

| J | M | If J, M |
|---|---|---|
| T | T | T |
| F | F | T |
| F | F | F |

J and M may both be false, and if they are, some further condition has to be satisfied for 'If J, M' to be true. Perhaps the further question, if John and Mary didn't go, is whether Mary would have gone if John had, and he can't be certain of that. Our positive account and Assumption 4 diverge, and intuition, once more, favours our account. Assumption 4 must be rejected. Putative possibility 4b does not exist.

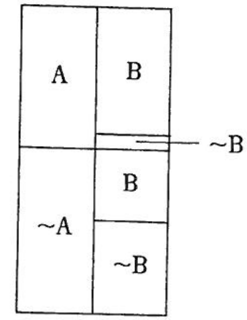
We have reached the end of our proof. That the conditional has non-truth-functional truth-conditions entails that at least one of Assumptions 1 to 4 is true. But whichever we take, we can find conditionals whose acceptability (or unacceptability), both intuitively and in terms of our positive account, conflicts with that assumption.

Given truth-conditions, we have a paradox. It is no accident that, given truth-conditions, there is philosophical disagreement about whether or not they are truth-functional. For there are acceptable conditionals whose acceptability cannot be accommodated by any non-truth-functional account. I have used some of these in the above proof. And there are unacceptable conditionals whose unacceptability cannot be accommodated by the truth-functional account. I used these earlier in the case against truth-functionality. But our positive account resolves this paradox. The mistake is to think of conditionals as part of fact-stating discourse.

Perhaps we can get closer to the heart of the paradox with the following case. I am wondering whether A and whether B. Someone comes along who knows their truth-values, but feels unable to tell me all he knows. He says 'The most I am able to tell you is this: ~(A & ~B).' This is enough for me to conclude that if A, B. Now, ~(A & ~B) *does not entail* 'If A, B'. That is the truth-functional account, with all its difficulties. But *belief that* ~(A & ~B) *in the absence of belief that* ~A *is sufficient for belief that if A, B* (see Figure 6). No non-truth-functional truth-conditions can accommodate that fact.[10]



FIG. 6




. . .

## 7. SOME CONCLUDING OBSERVATIONS

The argument makes no assumptions about what truth consists in—beyond the fact that one may take various epistemic attitudes to the question whether a given proposition has that property. Whatever 'true' means, to judge it likely that it applies to B on the assumption that it applies A is not equivalent to judging it likely that it applies to something else. The linguistic or mental act of *supposing* is ineliminable from conditionals, and they cannot be reduced to straight assertions or beliefs.

Another way of putting the conclusion is this. One can be certain or uncertain about a proposition, A. Uncertainty about A (~A, A ∨ B, etc.) has a structure which is not only compatible with the proposition's having one or other truth-value, but requires that it does. One can be certain or uncertain about whether if A, B. Uncertainty about a conditional has a structure which does not require that the conditional has one or other truth-value; moreover, it is incompatible with this.

There are several reasons why this argument is important. This is

10. This sentence conflicts with the thesis of Robert Stalnaker's "Indicative Conditionals" (1975). That paper and this one are both included in Jackson (1991), and the point at issue is discussed on pp. 198–99 of that version of my essay.

the most general one: a hard argument against (or for) the applicability of the concept of truth to a given area of discourse is a rare thing. It is just possible that this one may shed light on controversies about the applicability of the concept in other areas. Given certain key features of the epistemology of discourse of the kind of question, we can ask, does this epistemology fit with even a minimal metaphysics of truth?

Another reason why the consequences of the argument are far-reaching is that it has become increasingly fashionable to 'analyse' other important philosophical concepts in terms of conditionals, for example, causation, natural laws, dispositional properties, and more recently, knowledge. The standard account of statements of the form 'All A's are B' is also a striking example. There is much that needs to be re-examined in the light of this thesis.

Perhaps most importantly, the criterion for the validity of deductive arguments needs to be restated in the light of this thesis. The standard criterion is that valid arguments preserve truth. But such arguments contain conditionals, and according to the thesis I have defended, conditionals are not suitable candidates for truth. Now, our interest in the validity of arguments is epistemological. A valid argument is one such that it is irrational to accept the premisses and reject the conclusion. Construing acceptance as high subjective probability, and acceptance of a conditional in terms of high conditional probability, Adams has shown how to give a precise criterion of validity along these lines, which coincides with the standard one for arguments without conditionals.[11] It explains why certain patterns of inference involving conditionals are valid; and it isolates the unusual conditions under which others, which appear valid, fail. I discussed one such example at the end of Sect. 5.

Finally, this argument should not be construed as part of a general attack on truth-conditional semantics. It depends on a contrast between the roles of the constituent sentences of a conditional and the conditional itself. It does not require, but fits well with a truth-conditional account of our understanding of the former.

Indeed, this anti-realist argument about conditionals is more puzzling for a general anti-realist than for a philosopher with strong realist tendencies. For the latter, let us say, a declarative sentence identifies a possible state of affairs. It is true if and only if the state of

11. See Adams (1975), ch. 2. It is worth remarking that the existence of logical consequences of moral judgements, rules, laws, etc. also suggests that the classical account of validity is limited in scope.



affairs identified obtains. For him, the argument shows that there are no conditional states of affairs. For an anti-realist who construes truth along the lines of what is ideally rationally acceptable, it is much more puzzling that the notion cannot be applied to conditionals. But, as I said before, the argument itself makes no assumptions about the nature of truth.[12]

12. Earlier versions of this essay were read to the Oxford Philosophical Society in 1984 and the Conference on the Philosophy of Logic and Language in Leicester, 1985. It formed part of the material of a lecture course on Conditionals given at the Instituto de Investigacions Filosóficas, Universidad Nacional Autónoma de México in the summer of 1985. I am grateful to these audiences and many other people for useful comments, and especially to Raúl Orayen for his enthusiasm and constructive criticism.

# A Philosophical Companion to First-Order Logic

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